Keller cones in aeroacoustics

C. J. Chapman

Keele University

Presentation given at the Isaac Newton Institute,
Cambridge University

'Mathematics in the Spirit of Joe Keller'

2 March 2017

The geometrical theory of diffraction

- This talk is about the Geometrical Theory of Diffraction.
- It was invented in the 1940–50's by Joe Keller, and has been called 'the greatest advance in optics since Newton'.
- Keller wrote a review paper on the subject ('JOSA 1962') which has become a classic, and has been cited many thousands of times.
- This paper, which besides the theory includes canonical diagrams of various cones and special cases such as a disc, has influenced innumerable applied mathematicians, physicists, and engineers (including CJC).

CJC contact with Joe Keller

- First occasion was the Woods Hole GFD programme.
- This could be true for several DAMTP fluid dynamicists and members of the audience, judging by the WHOI photographs. Joe Keller was a prominent member of this programme for decades. (Beards came and went, including Joe's.)
- Fellows and visiting scientists who would have met Joe there include MEM, EJH, MREP, PFL, PHH, HEH, NOW, DG, CPC, PAM, DV, PJD, and no doubt others.
- We had further contact during Joe's extended visit to DAMTP in early 1990's, during which we often discussed ray theory. He took a kindly interest in my work.
- I soon found that Joe remembered what I said, and on one occasion he came back with a calculation relating to one of my remarks.

Influence on aeroacoustics

- ‘Keller cones’ are now mainstream in aeroacoustic theory, and appear (explicitly or implicitly) in the work of
  - CJC, Hocter, Powles (duct diffraction, caustics, hyperbolic umbilic catastrophe, leading-edge noise);
  - NP, Kerschen, Graeme Keith, Raphael Assier, Lorna Ayton, . . . (aeroengines, turbomachinery);
  - Amiet, Glegg, Moreau, IDA, David Nigro,. . . (blade-vortex interaction);
  - Many others, e.g. aeroacoustics researchers at ISVR (Southampton), Rolls-Royce (Derby), ...

A classic paper

Let's begin by looking at Keller's 1962 paper (in reverence, rather than to read today!). Keller had a distinctive writing style—short clear sentences, with displayed equations almost invariably at the end of sentence. This style is not so easy to imitate.

KN JOSA 'The classical paper on GTD' 62.52 116-130 J. B. KELLER

JOU . OF THE OPTICAL SOCIETY OF AMERICA VOLUME 52, NUMBER 2 FEBRUARY, 1962

~~JOSA~~ NEW YORK

Geometrical Theory of Diffraction* (1962)

JOSEPH B. KELLER
Institute of Mathematical Sciences, New York University, New York, New York

(Received September 13, 1961)

The geometrical theory of diffraction is an extension of geometrical optics which accounts for diffraction. It introduces diffracted rays in addition to the usual rays of geometrical optics. These rays are produced by incident rays which hit edges, corners, or vertices of boundary surfaces, or which graze such surfaces. Various laws of diffraction, analogous to the laws of reflection and refraction, are employed to characterize the diffracted rays. A modified form of Fermat's principle, equivalent to these laws, can also be used. Diffracted wave fronts are defined, which can be found by a Huygens wavelet construction. There is an associated phase or eikonal function which satisfies the eikonal equation. In reference point. The amplitude varies in accordance with the principle of conservation of energy in a narrow tube of rays. The initial value of the field on a diffracted ray is determined from the incident field with the aid of an appropriate diffraction coefficient. These diffraction coefficients are determined from certain canonical problems. They all vanish as the wavelength tends to zero. The theory is applied to diffraction by an aperture in a thin screen diffraction by a disk, etc., to illustrate it. Agreement is shown between the predictions of the theory and various other theoretical analyses of some of these problems. Experimental confirmation of the theory is also presented. The mathematical justification of the

The Keller cone of diffracted rays

Here is the famous diagram of the cone and disc.

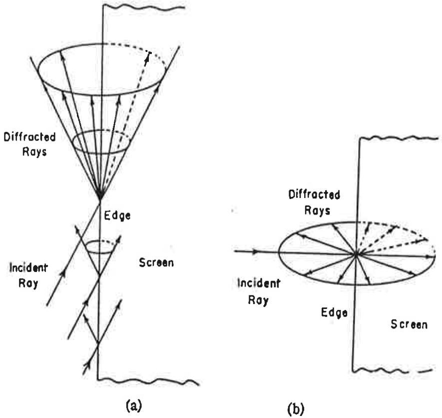


FIG. 1. (a) The cone of diffracted rays produced by an incident ray which hits the edge of a thin screen obliquely. (b) The plane of diffracted rays produced by a ray normally incident on the dge of a thin screen.

integral asymptotically for short wavelengths when th incident field was a spherical wave, by using the Mag transformation. He showed that the diffracted field at point Q consisted of contributions from a small numbe of points on the edge. If we draw straight lines fron these points to Q and call them diffracted rays we fin that all of them at smooth parts of the edge satisfy th law of edge diffraction. The integrals of the Kirchho and modified Kirchhoff method, which employs Ray leigh's formulas, have been evaluated asymptotically fo short wavelengths by van Kampen[5] and by Keller *et al.* for arbitrary incident fields. The latter authors also evaluated Braunbek's[7] improved version of the Kirch hoff integrals. In all cases the points on smooth parts o the edge which contribute to the diffracted field corre spond to diffracted rays satisfying the law of edge diffraction.

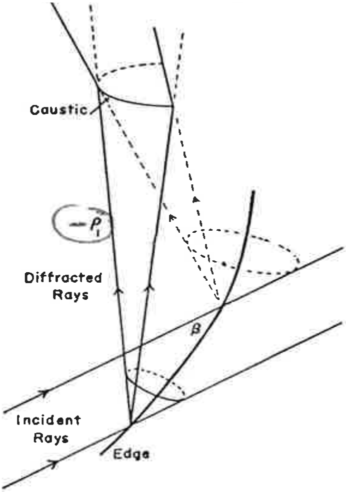
An indirect experimental verification of the existence of edge diffracted rays and of the law of edge diffr

Caustics

Another diagram from JOSA 1962 shows neighbouring cones intesecting at a caustic.

FIG. 8. A pair of neighboring incident rays hitting a curved edge, and some of the resulting diffracted rays. The two cones of diffracted rays intersect at the caustic, which is at the distance ρ_1 from the edge along the rays.

Should be $-\ell_1$



Application to duct aeroacoustics (1994)

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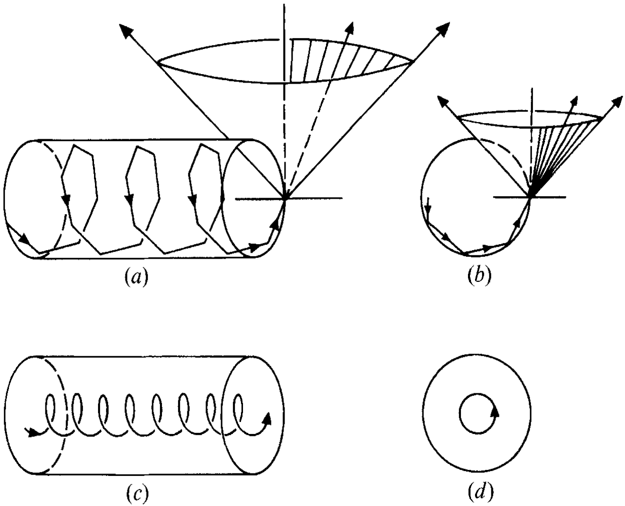
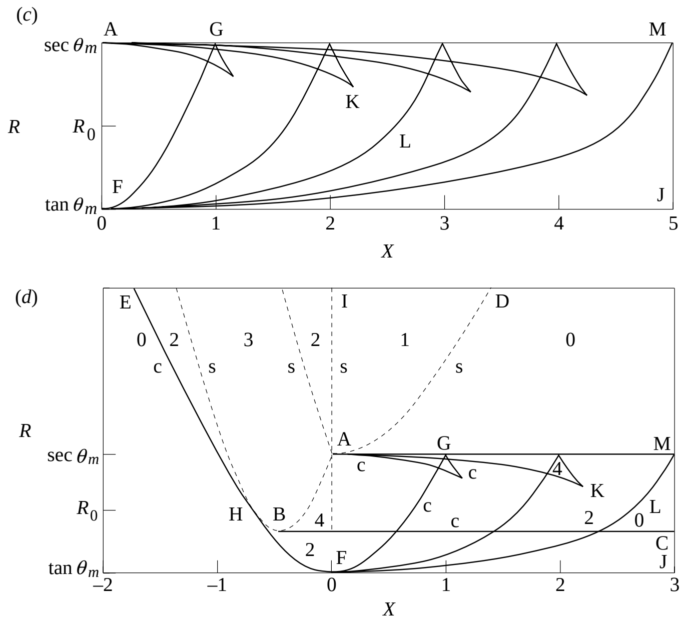


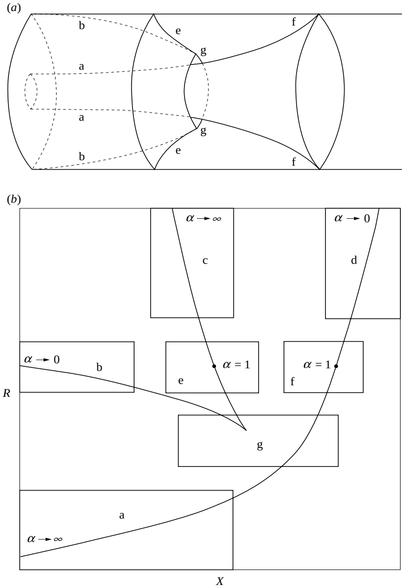
Explanatory power of the 'duct cones'

- In the forward arc: rays from two cones pass through an observation point. This explains the multi-lobed interference pattern in this region.
- In the rear arc: either one ray or no ray passes through an observation point (because the duct wall is in the way). This explains the single very broad lobe, up to a 'cliff edge' at an angle determined by the vertex angle of the cone.
- Exactly one-quarter of the rays on a cone point back into the duct, and constitute the reflected field. This is a striking geometrical explanation of 'where the reflected field comes from'.
- The reflected field has an intricate caustic structure, determined globally by a hyperbolic umbilic catastrophe.

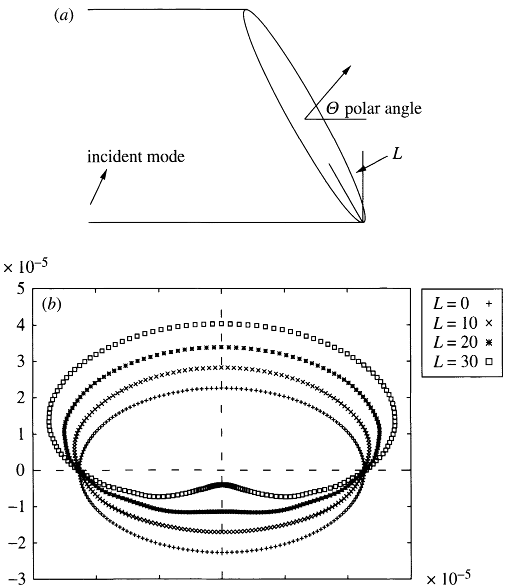
Hyperbolic umbilic in duct (1999) (cf. IDA, ...1994)



Hyperbolic umbilic in duct (1999)



Scarfed duct (2004) (NP, G. Keith)



Blade-vortex interaction (Amiet)

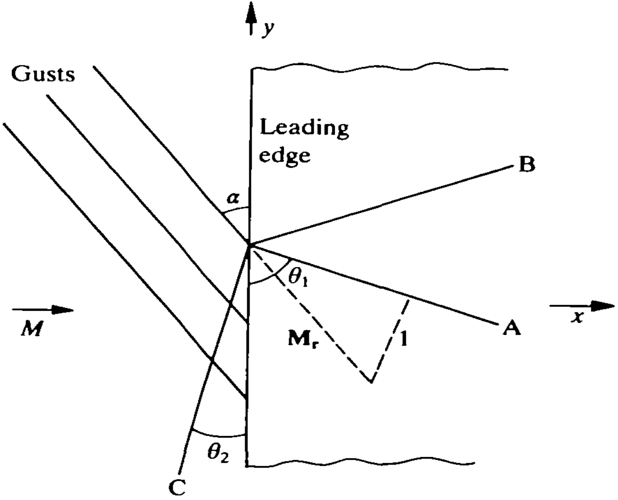


Fig. 6. Gust interaction with the leading edge

Recent very detailed work (2013) (Lorna Ayton)

Outer regions have ray structure described by Keller cones or discs.